Measurement of Turbulence Intensities with Piezoelectric Probes in Viscoelastic Fluids

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The piezoelectric impact probe of Wells et al. was used to compare measurements of turbulence intensity in a pipe of purely viscous solvents with viscoelastic polymer solutions and to compare with measurements made with hot-film anemometer probes. An equation was derived which shows the form of normal and shear stress contributions to measured fluctuating impact pressure in viscoelastic fluids. The equation indicated that for viscoelastic fluids, low values of intensities of turbulence can be obtained by neglecting normal and shear stress effects. Experimental measurements showed that turbulence intensities measured at the pipe center with the piezoelectric probe were the same as those measured with a hot-film wedge probe for viscoelastic solutions, but that as the wall was approached, the piezoelectric probe values were low for the viscoelastic solutions.

Nomenclature

e_x = unit vector in the axial direction in cylindrical coordinates
$\tilde{e}_r$ = unit vector in the radial direction in cylindrical coordinates
$\tilde{e}_\phi$ = unit vector in the tangential direction in cylindrical coordinates
$g(x)$ = function of the shear stress fluctuations
$\tilde{l}$ = displacement vector
p' = fluctuating component of pressure
P = pressure
$\bar{P}$ = time-averaged value of P
P_p = pressure measured by impact tube
p_t' = fluctuating component of pressure measured by piezoelectric probe
P_t = total pressure measured by piezoelectric probe
$\bar{P}_t$ = time-averaged value of P_t
P_w = static pressure at the wall
r = radial coordinate
R = tube radius
u' = fluctuating velocity component in the axial direction
U = velocity in the axial direction
$\bar{U}$ = time-averaged value of U
v' = fluctuating velocity component in the radial direction
$\tilde{V}$ = velocity vector in cylindrical coordinates
x = longitudinal coordinate
λ_c = characteristic time of the flow
ρ = density
μ = viscosity
σ_{ii} = deviatoric normal stress
$\overline{\sigma_{ii}}$ = time-averaged deviatoric normal stress
σ_{ii}' = fluctuating deviatoric normal stress
τ = characteristic solution relaxation time
τ_w = shear stress at the wall
τ_{ij} = shear stress
$\bar{\tau}_{ij}$ = time-averaged value of shear stress
τ_{ij}' = fluctuating shear stress
$\langle\ \rangle$ = root mean square of enclosed quantity

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Introduction

WELLS and co-workers[1,2] have used piezoelectric probes to measure turbulence intensities and energy spectra in purely viscous fluids and for drag reducing polymer solutions in pipe flow. The piezoelectric probes consist of a piezoelectric sensor mounted within an impact tube and they measure total head pressure fluctuations. For a purely viscous fluid, total head pressure fluctuations can be directly related to velocity fluctuations using the equation

$$\langle u'\rangle/\bar{U} = \langle p_t'\rangle/\rho\bar{U}^2 \tag{1}$$

where $\langle u'\rangle$ is the root-mean-square value of velocity fluctuations in the longitudinal direction, $\bar{U}$ is the local mean velocity and $\langle p_t'\rangle$ is the root-mean-square value of the pressure fluctuations sensed by the piezoelectric probe. Equation (1) was derived by neglecting the normal and shear stress terms in the equations of motion.

For drag reducing viscoelastic polymer solutions, mean velocity profile measurements in pipe flow obtained using impact tubes have demonstrated that large normal stress effects exist for more concentrated solutions and that mean velocities are lower than the actual velocities.[3–8] Astarita and Nicodemo[3] have shown that

$$P_p - P_w = -(\sigma_{xx} - \sigma_{rr}) - \int_r^R \frac{\sigma_{rr} - \sigma_{\phi\phi}}{r}\, dr + \frac{1}{2}\rho\bar{U}^2 \tag{2}$$

where P_p and P_w are the static pressure measurements at the impact tube and at the wall, and σ_{ii} are deviatoric normal stresses. Thus, for many drag reducing viscoelastic fluids, normal stresses cannot be neglected and the question arises as to whether viscoelasticity will affect the measurements obtained with a piezoelectric probe.

The objects of this paper are 1) to extend the analysis of Wells et al. to include normal and shear stress effects, obtaining a more general form for Eq. (1); 2) to present intensity of turbulence data obtained with a piezoelectric probe in solutions of polyisobutylene (PIB) in mineral oil; and 3) to analyze the intensity data obtained in the PIB in mineral oil solutions and all intensity data reported in the literature for piezoelectric probes and to compare the trends observed in these data with those reported for intensity data obtained using hot-film anemometry.

Generalization of the Wells, Harkness and Meyer Equation

The development in this section follows closely the original work of Wells et al.,[1] and some of the steps in the derivation which are presented in detail in the original reference will be omitted. Wells et al. considered the equation of motion in the form

$$D\tilde{V}/Dt = -(1/\rho)\,\text{grad}\,P \tag{3}$$

assuming no shear stresses and no deviatoric normal stresses and an incompressible, nonconducting fluid. To Eq. (3), the deviatoric normal stress and shear stress contributions are added:

$$\frac{D\tilde{V}}{Dt} = -\frac{1}{\rho}\,\text{grad}\,P + \frac{1}{\rho}\left[\frac{\partial\sigma_{xx}}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rx}) + \frac{1}{r}\frac{\partial\tau_{x\phi}}{\partial\phi}\right]\tilde{e}_x + \frac{1}{\rho}\left[\frac{\partial\sigma_{rr}}{\partial r} + \frac{(\sigma_{rr}-\sigma_{\phi\phi})}{r} + \frac{1}{r}\frac{\partial\tau_{r\phi}}{\partial\phi} + \frac{\partial\tau_{rx}}{\partial x}\right]\tilde{e}_r + \frac{1}{\rho}\left[\frac{1}{r}\frac{\partial\sigma_{\phi\phi}}{\partial\phi} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\phi}) + \frac{\partial\tau_{x\phi}}{\partial x}\right]\tilde{e}_\phi \tag{4}$$

where $\tilde{e}_r$, $\tilde{e}_\phi$, and $\tilde{e}_x$ are unit vectors in cylindrical coordinates. Defining $d\tilde{l} = (dx)\,\tilde{e}_x + (dr)\,\tilde{e}_r + (d\phi)\,\tilde{e}_\phi$ and $\tilde{V} = U\tilde{e}_x + V\tilde{e}_r + W\tilde{e}_\phi$, and taking the dot product of $d\tilde{l}$ with Eq. (4), one obtains

$$\frac{D\tilde{V}}{Dt}\cdot d\tilde{l} = -\frac{1}{\rho}\,\text{grad}\,P\cdot d\tilde{l} + \frac{1}{\rho}\left[\frac{\partial\sigma_{xx}}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rx}) + \frac{1}{r}\frac{\partial\tau_{x\phi}}{\partial\phi}\right]dx + \frac{1}{\rho}\left[\frac{\partial\sigma_{rr}}{\partial r} + \frac{(\sigma_{rr}-\sigma_{\phi\phi})}{r} + \frac{1}{r}\frac{\partial\tau_{r\phi}}{\partial\phi} + \frac{\partial\tau_{rx}}{\partial_x}\right]dr + \frac{1}{\rho}\left[\frac{1}{r}\frac{\partial\sigma_{\phi\phi}}{\partial\phi} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\phi}) + \frac{\partial\tau_{x\phi}}{\partial x}\right]d\phi \tag{5}$$

Equation (5) is written in the form of a line integral over a path c. The integration path is chosen as

$$d\tilde{l} = (dx)\tilde{e}_x + (0)\tilde{e}_r + (0)\tilde{e}_\phi \tag{6}$$

Wells et al. indicate that their integration was performed along a streamline, as

$$\int_c \frac{D\tilde{V}}{Dt}\cdot d\tilde{l} = \int_c \frac{\partial\tilde{V}}{\partial t}\cdot d\tilde{l} + \int_c \tilde{V}\,\text{grad}\,\tilde{V}\cdot d\tilde{l} = -\frac{1}{\rho}\int_c \text{grad}\,P\cdot d\tilde{l}$$

Neglecting the term $\partial\tilde{V}/\partial t\cdot d\tilde{l}$ based on an order of magnitude analysis, the preceding equation is integrated to

$$\tfrac{1}{2}\tilde{V}^2 + (1/\rho)P = \text{const}$$

This is true only if the integration is performed along a streamline. The integration path used in this paper is not along a streamline since the differential element of length $(dx, 0, 0)$ is not tangent to the instantaneous velocity vector at every point in space. The terms involving the normal and shear stress terms cannot be written in the form grad $A\cdot d\tilde{l} = dA$, as occurs for grad P and grad $\tilde{V}^2$ in Wells' analysis, and thus it becomes necessary to specify the integration path as $(dx, 0, 0)$. Since dr and $d\phi$ equal 0, the integral of Eq. (5) can be written as:

$$\int_c\left[\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial r} + \frac{W}{r}\frac{\partial U}{\partial\phi}\right]dx = \frac{1}{\rho}\int_c\left[\frac{\partial\sigma_{xx}}{\partial x} - \frac{\partial P}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rx}) + \frac{1}{r}\frac{\partial\tau_{x\phi}}{\partial\phi}\right]dx \tag{7}$$

The left-hand side of Eq. (7) can be rewritten, after neglecting the term $\partial U/\partial t$, as

$$\int_c\left[U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial r} + \frac{W}{r}\frac{\partial U}{\partial\phi} + V\frac{\partial V}{\partial x} - V\frac{\partial V}{\partial x} + W\frac{\partial W}{\partial x} - W\frac{\partial W}{\partial x}\right]dx = \int_c\left[\frac{1}{2}\frac{\partial\tilde{V}^2}{\partial x} + V\left(\frac{\partial U}{\partial r} - \frac{\partial V}{\partial x}\right) + W\left(\frac{1}{r}\frac{\partial U}{\partial\phi} - \frac{\partial W}{\partial x}\right)\right]dx$$

The terms $V(\partial U/\partial r - \partial V/\partial x)$ and $W[(1/r)(\partial U/\partial\phi) - \partial W/\partial x]$ are equal to zero when the integration is performed along a streamline. In this analysis, they must remain in the equation since the integration path is $(dx, 0, 0)$. The integral terms are a function of x alone, since r and ϕ were held constant along the integration path. For turbulent flow,

$$U = \bar{U} + u',\quad V = \bar{V} + v',\quad W = \bar{W} + w',\quad P = \bar{P} + p'$$

$$\tau_{ij} = \bar{\tau}_{ij} + \tau_{ij}',\quad \sigma_{ii} = \bar{\sigma}_{ii} + \sigma_{ii}'$$

With $\bar{V} = \bar{W} = 0$, Eq. (7) can be integrated to

$$\frac{1}{2}\tilde{V}^2 + \int_c\left[v'\left(\frac{\partial\bar{U}}{\partial r} + \frac{\partial u'}{\partial r} - \frac{\partial v}{\partial x}\right) + w'\left(\frac{1}{r}\frac{\partial u'}{\partial\phi} - \frac{\partial w'}{\partial x}\right)\right]dx = -\frac{\bar{P}}{\rho} - \frac{p'}{\rho} + \frac{\sigma_{xx}}{\rho} + \frac{\sigma_{xx}'}{\rho} + \frac{1}{\rho}\int_c\left[\frac{1}{r}\frac{\partial}{\partial r}(r\bar{\tau}_{rx}) + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rx}') + \frac{1}{r}\frac{\partial\tau_{x\phi}'}{\partial\phi}\right]dx + C \tag{8}$$

using the symmetry conditions, $\partial\bar{U}/\partial\phi = 0$, $\partial\bar{\tau}_{x\phi}/\partial\phi = 0$. If one neglects the integral terms and $\bar{\sigma}_{xx}$ and σ_{xx}', the resulting equation is the same as that obtained by Wells et al.:

$$\tfrac{1}{2}\tilde{V}^2 = -P/\rho - p'/\rho + C$$

If only the normal and shear stress terms are neglected, Eq. (8) becomes

$$\frac{1}{2}\tilde{V}^2 + \int_c\left[v'\left(\frac{\partial\bar{U}}{\partial r} + \frac{\partial u'}{\partial r} - \frac{\partial v'}{\partial r}\right) + w'\left(\frac{1}{r}\frac{\partial u'}{\partial\phi} - \frac{\partial w'}{\partial x}\right)\right]dx = -\frac{P}{\rho} - \frac{p'}{\rho} + C \tag{9}$$

The difference between this equation and Wells' equation is the integral term, which occurs because the integration path is not along a streamline. In both cases, the constant of integration is $\bar{P}_t + p_t'$, since the integral term in Eq. (9) is evaluated within limits and at the stagnation point, $\tilde{V} = 0$, $\bar{P} = \bar{P}_t$ and $p' = p_t'$. After using the equation of continuity in Eq. (9), time-averaging, subtracting the time-averaged equation, and integrating $v'\partial v'/\partial x$ and $w'\partial w'/\partial x$, one obtains

$$u'\bar{U} = \left(\frac{p_t' - p'}{\rho}\right) + \frac{1}{\rho}\int_c\left\{v'\left(\frac{\partial\bar{U}}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial r}[r(u'v' - \overline{u'v'})] + \frac{1}{r}\left[\frac{\partial}{\partial\phi}(u'w' - \overline{u'w'})\right]\right\}dx \tag{10}$$

The term $\tilde{V}^2$ equals $(\bar{U} + u')^2 + v'^2 + w'^2$, and u'^2, v'^2, w'^2 were neglected when compared to $\bar{U}^2$.

The integral term in Eq. (10) does not depend on the rheological properties of the fluid and should be of the same order of magnitude for both viscoelastic and purely viscous fluids. Experimental results indicate that Wells' equation,

$$\langle u'\rangle/\bar{U} = \langle p_t'\rangle/\rho\bar{U}^2$$

has been successful for calculating intensities of turbulence in purely viscous fluids. For purely viscous fluids, the integral term in Eq. (10) can be neglected when compared to p_t', and the same holds for viscoelastic fluids.

For viscoelastic fluids, the time-dependent normal and shear stress terms in Eq. (8) depend strongly on the rheological properties of the fluid and on the deformation history to which an element of fluid is subjected. These terms are the ones that are of interest when trying to analyze the effects of viscoelasticity on intensity measurements in viscoelastic fluids.

Equation (8) can be written, after time-averaging and subtracting the time-averaged equation from the original equation, as

$$u'\bar{U} = \frac{1}{\rho}(p_t' - p' + \sigma_{xx}') + \frac{1}{\rho}\int_c \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau'_{rx}) + \frac{1}{r}\frac{\partial \tau'_{\phi x}}{\partial \phi}\right] dx \quad (11)$$

The constant of integration in Eq. (8) is $P_t + p_t'$. The integral term will be represented as $g(x)$. The turbulence intensity can be obtained from Eq. (11):

$$\langle u'\rangle/U = \{\langle p_t'\rangle^2 + \overline{2p_t'[\sigma_{xx}' - p' + g(x)]} + \langle \sigma_{xx}' - p' + g(x)\rangle^2\}^{1/2}/\rho\bar{U}^2 \quad (12)$$

The term $\overline{p_t'[\sigma_{xx}' - p' + g(x)]}$, which represents the correlation of random fluctuations at a distance x, is assumed to be zero. Thus,

$$\frac{\langle u\rangle}{\bar{U}} = \frac{[\langle p_t'\rangle^2 + \langle \sigma_{xx}' - p' + g(x)\rangle^2]^{1/2}}{\rho\bar{U}^2} \quad (13)$$

Equation (13) is the analog of Eq. (1), obtained considering all terms in the equation of motion. This form is not useful for the calculation of turbulence intensities, but it is important, since it is clear that intensities of turbulence calculated using Eq. (1) will be lower than actual values when viscoelastic effects are important. Equation (1) would be valid only if the effects of σ_{xx}', p', and $g(x)$ cancelled each other. There is no basis for making this assumption. It is not possible, with the present knowledge of turbulent flow of viscoelastic fluids, to predict the magnitude of the error resulting from their neglect. However, as will be seen below, it appears that the error introduced is negligible at the center of the tube, but becomes significant close to the wall.

For purely viscous fluids, the value of $\overline{\sigma_{xx}}$ is small and is given by Ref. 3:

$$\overline{\sigma_{xx}} = \overline{\rho u'^2} + 2\mu(\nabla\cdot\tilde{V}) \quad (14)$$

For an incompressible Newtonian fluid, $\nabla\cdot\tilde{V} = 0$. Even near the wall where the intensity of turbulence is high, the value of $\overline{\rho u'^2}/\rho\bar{U}^2$ is approximately 0.10. For viscoelastic fluids, Eq. (14) does not apply since the stresses cannot be represented by the Newtonian relationship. The value of $\overline{\sigma_{xx}}$ in a viscoelastic fluid is greater than in purely viscous fluids, and very near the wall it may be quite large. It is reasonable to assume that the normal stress fluctuations σ_{xx}' will also be larger than for purely viscous fluids, especially near the wall. Shear stress fluctuations in viscoelastic fluids may also be much larger than for purely viscous fluids since the very high instantaneous deformation rates to which the fluid is subjected may cause nonlinear viscoelastic effects such as stress overshoots, which will lead to large stress fluctuations. Thus, both σ_{xx}' and $g(x)$ in viscoelastic fluids are larger than for purely viscous fluids.

It is important to notice that errors caused by the use of Eq. (1) are not due to an erroneous response of the piezoelectric probe. The probe reflects p_t' values but these may be only a portion of the total intensity. Errors in the predicted intensity are due to neglected contributions of σ_{xx}', p', and $g(x)$ to u at a given position in the tube.

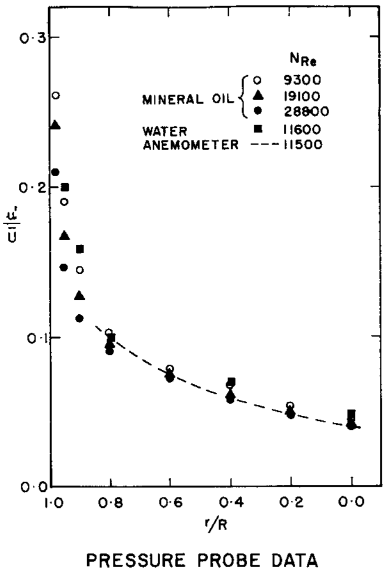

Fig. 1 Pressure probe data for mineral oil and water compared with hot-film anemometer data.

Experimental Results

Intensity of turbulence measurements were obtained in three solutions (0.028, 0.05, and 0.20% by weight) of polyisobutylene L-200§ in mineral oil in a 1-in. i.d. tube. The pumping unit used for this work has been described by Patterson and Zakin[9] and by Hershey and Zakin[10] in considerable detail.

The piezoelectric probe system, consisting of a total pressure probe and a static pressure probe, was similar to the one described by Wells et al.[1] and Spangler[2] and was provided by Ling-Temco-Vought, Inc. (LTV). The pressure probe measures the rms values of pressure fluctuations p_t'. Intensities of turbulence were calculated using Eq. (1).

Wells et al.[1] and Spangler[2] have already shown that the piezoelectric probe measures turbulence intensities in water correctly. Figure 1 shows intensity profile results obtained with the piezoelectric probe system with mineral oil at several Reynolds numbers. Mineral oil data at $N_{Re} = 9300$ agree very well with the water data obtained in a $\frac{3}{4}$-in. i.d. tube, and with hot-film anemometry results for pure liquids and air[9,11] at about the same Reynolds numbers. There is a trend to lower intensities as Reynolds numbers increase. Mean velocity profiles were not measured, but were predicted using the law of the wall. The results of Fig. 1 for oil and water show that very little error was introduced by this method of predicting velocity profiles.

Velocity profiles for polymer solutions were predicted in the same manner. No corrections for viscoelastic effects on the velocity profiles were made as the appropriate correction terms cannot be evaluated. However, from measurements reported by Patterson and Florez,[4] it has been established that velocities at the center of the tube are somewhat higher than those predicted for purely viscous fluids. Thus, intensities at the center of the tube calculated using Eq. (1) with the predicted velocity profiles will be greater than those that would be obtained with the actual velocity profiles. The maximum error is of the order of 20% (based on Florez's

§ Enjay Chemical Company, $M_v \approx 4.7 \times 10^6$.

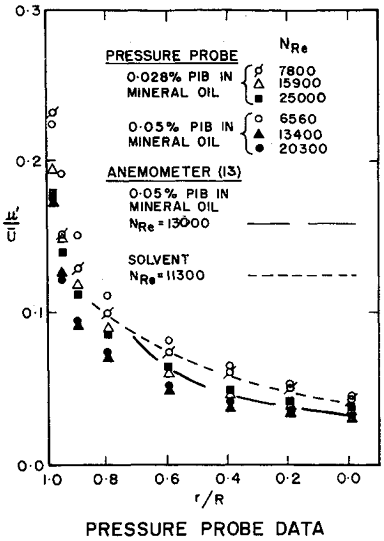

Fig. 2 Pressure probe data for viscoelastic polymer solutions (0.028% and 0.05% PIB in mineral oil) compared with hot-film anemometer data.

results) at $r/R = 0.00$ and decreases to zero at r/R close to 0.60, where the profiles for purely viscous fluids and polymer solution cross. Nearer the wall, the estimation of the velocity profile will lead to prediction of low intensities of turbulence with the maximum error at $r/R = 0.80$ being of the order of -10%. The error close to the viscous sublayer (beyond $r/R = 0.90$) can be very large because of the uncertainty in predicting velocities in this region.

Figure 2 shows piezoelectric probe intensity profile results for 0.028 and 0.05% PIB solutions in mineral oil at several Reynolds numbers. The 0.028% solution gave normal (solvent) intensities at 7800 Reynolds number and lower than normal intensities at higher Reynolds numbers. The 0.05% solution gave intensities slightly above normal as the wall was approached ($r/R = 0.8$) at the lowest Reynolds number and considerably lower than normal at all positions at the higher Reynolds numbers. Intensities close to the wall are subject to the errors introduced by the predicted velocity profiles as noted previously and valid comparisons cannot be made.

At $N_{Re} = 13{,}000$, the wedge hot-film anemometer results for the 0.05% solution are very close to those obtained with the pressure probe at the tube center at the same Reynolds number. However, the intensity rises rapidly as the wall is approached and is considerably higher than that of the pressure probe for this solution and Reynolds number at $r/R = 0.7$.

Figure 3 shows pressure probe data for 0.20% PIB in mineral oil solution. This higher concentration solution is more viscoelastic than the 0.028 and 0.05% solutions, and the trends observed in this solution are different from those observed in the two other PIB solutions studied with the piezoelectric probe (see Analysis of Results). At $r/R = 0.00$, the intensity at $N_{Re} = 3500$ is 0.045; at $N_{Re} = 6500$ and 10,000 the intensity increased to 0.07 (data indicated by START). Measurements taken after approximately one and a half hours of pumping at the same flow rate are indicated by END in Fig. 3. The high values of intensities observed at the beginning of the runs have all decreased in value. This degradation effect has also been observed for other solutions studied.[11–13]

All of the data in Fig. 3 were taken on the same solution. To minimize the effect of degradation on data for the next Reynolds number to be run, the profile for the lowest Reynolds number was run first, and the highest was run last. At all positions in the core region (to $r/R = 0.8$), intensity levels were in the same order as Reynolds numbers.

Analysis of Results

The results obtained with the piezoelectric probe are compared below to measurements obtained on viscoelastic, drag reducing fluids using a hot-film wedge probe with a constant temperature anemometer system.[11–13] Other measurements of intensities of turbulence in drag reducing fluids have been reported,[6,14–16] but they do not cover a sufficiently wide range of concentrations and polymer systems to give an over-all picture of the intensity patterns. Frequency response measurements for the wedge hot-film probe in viscoelastic fluids have been performed[12,17] and indicate that intensities of turbulence obtained in viscoelastic fluids are correct.

Centerline Intensity Measurements

Results obtained at the center of the tube ($r/R = 0.00$) for a large number of polymer and soap solutions with hot-film anemometry are illustrated qualitatively in Fig. 4 as turbulence intensity vs a Deborah number.[13] This Deborah number is defined as the ratio of a relaxation time of the fluid to a characteristic time of the flow. The relaxation times of a fresh solution are larger than those of a degraded solution and those of a concentrated solution are greater than those of a dilute solution. For a given tube, the characteristic time of the flow is inversely proportional to velocity. Thus, dilute, degraded solutions pumped at low velocities correspond to small Deborah numbers. Fresh, concentrated solutions at high velocities correspond to large Deborah numbers. Region D-E corresponds to intensities increasing with increasing flow rate and reaching values higher than those for Newtonian fluids. Region B-C corresponds to intensities higher than normal at low flow rates which decrease with increasing flow rate and may reach values lower than normal. Region A-B corresponds to uniformly high intensities of turbulence.

There are three sets of intensity data obtained with piezoelectric probes in viscoelastic fluids: Spangler's[2] data for Polyhall P-295 (a polyacrylamide) in water; Wells, Harkness and Meyer's[1] data for CMC in water; and the PIB in mineral oil data reported here. In the vicinity of the center of the tube, where the local shear rates are very low, one would expect the viscoelastic and shear stress effects on the pressure probe to be small. From the experimental evidence, this appears to be the case. The intensity data for 0.01% P-295 in water reported by Spangler correspond to region A-B-C. A centerline intensity value of 17% was obtained at $N_{Re} \approx 10{,}000$; 16% was observed at $N_{Re} = 20{,}000$; and the intensity decreased to 3% at $N_{Re} = 200{,}000$ (Fig. 2 in Ref. 2) which is about the same as for water at this Reynolds number. Data for 0.0031% P-295 in water also appear to correspond to region A-B-C, although the intensity levels are lower—5.5–6% in the N_{Re} range 10,000–30,000 (corresponding water value is 4.0% at $N_{Re} = 20{,}000$). In the N_{Re} range 100,000–200,000 the intensities are lower than for water (Fig. 2 in Ref. 2). The data for two of the solutions reported here, 0.028 and 0.05% PIB in mineral oil, and also the 0.05% CMC in water studied by Wells et al. correspond to the region B-C (near the minimum). Data for the more viscoelastic 0.20% PIB in mineral oil solution correspond to region D-E (see Fig. 3) with intensities rapidly rising with increasing Reynolds (and Deborah) number. Degradation of the polymer molecules in this solution lowers the Deborah number, causing a decrease in intensity. These last results are particularly significant because they confirm trends in highly drag reducing solutions, which were first observed by Patterson[11] using hot-film anemometry and which were considered surprising. The trends have now been confirmed using two independent experimental techniques.

Thus, the trends in data at the center of the tube obtained using the impact probe and hot-film anemometry experimental techniques are the same. Although it is difficult, because of varying degradation effects on solutions (for instance, the intensity decrease in Fig. 3), to make direct quantitative comparisons, the comparison in the 0.05% PIB solution shows good agreement. Thus, it appears that viscoelastic and shear stress effects on the piezoelectric probe are negligible in the center of the tube, and Eq. (1) is applicable.

Intensity Profile Measurements

As the wall of the tube is approached, the local shear rate values become large, and viscoelastic and shear stress effects can become significant in the piezoelectric probe measurements.

Intensities for the 0.05% solution near the wall, measured by hot-film anemometry shortly after the pressure probe measurements, are shown in Fig. 2. They were lower than for the mineral oil. Intensities measured by the pressure probe for 0.028% and 0.05% PIB in mineral oil increase as the wall is approached but are even lower than for the hot-film measurements at an N_{Re} of 13,000 and higher. This discrepancy is probably caused by the viscoelastic effects noted previously.

In Spangler's viscoelastic 0.01% P-295 data, the intensity at $r/R = 0.90$ is only 10% compared to 16% at $r/R = 0.00$ at $N_{Re} = 2.39 \times 10^4$. At $N_{Re} = 1.51 \times 10^5$, the measured intensities near the wall are also extremely low: 8% (Fig. 3, in Ref. 2). For these two Reynolds numbers, there are apparent minima in the intensity profiles near the wall, which are very unusual. These are not observed in purely viscous or more dilute viscoelastic polymer solutions. For the 0.0031% P-295 solution, Spangler observed that normal stress effects on an impact tube for measuring mean velocity were small, with the integrated mass flow rates being less than 5% below the measured flow rates. For the 0.01% solution, the elastic effect was apparently quite large, giving integrated flow rates ranging from 15 to 29% lower than the measured values. It is interesting to note that Spangler calculated his intensities close to the wall based on the measured point velocities. If corrected velocities had been used the intensities for the 0.01% solution would have been even lower.

Studies of the response of the piezoelectric probes to known disturbances in the flow will have to be performed in order to determine quantitatively the level of viscoelastic effects since this is the only way to determine quantitative corrections to Eq. (1). Until these measurements are performed, intensities measured near the wall by a total head probe are questionable. Intensity measurements in the center of the tube, however, are quite reliable.

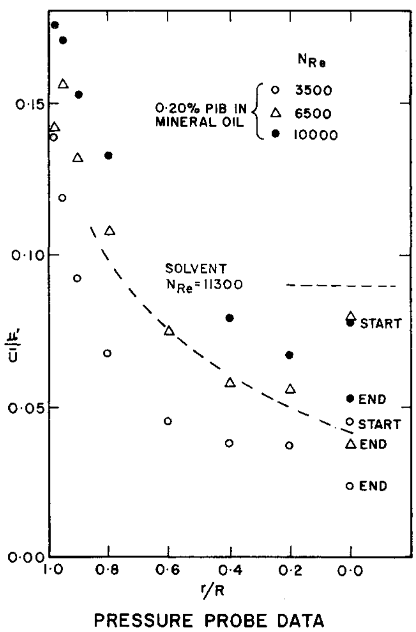

Fig. 3 Pressure probe data for viscoelastic polymer solution (0.2% PIB in mineral oil) compared with hot-film anemometer data for solvent.

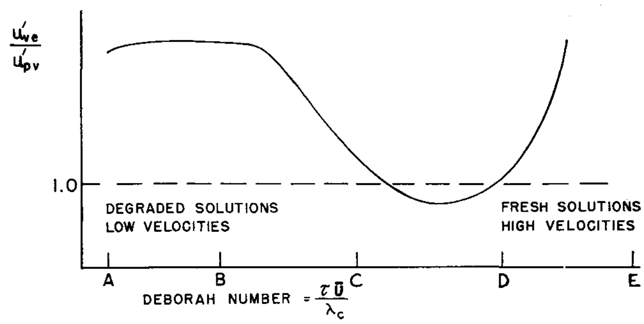

Fig. 4 Schematic representation of turbulence intensity vs Deborah number at the tube center for drag reducing solutions. Intensities of turbulence at $r/R = 0.000$ in viscoelastic, drag reducing solutions.

Conclusions

1) An equation has been derived that indicates that intensities of turbulence calculated using the equation given by Wells et al. for pressure probe measurements may be low when viscoelastic and shear stress terms that they did not consider are significant. Experimental results show that these terms are not significant for Newtonian fluids at any radial position.

2) Quantitative corrections cannot be made due to the lack of understanding of turbulent flow of viscoelastic fluids.

3) Measurements obtained at the center of the tube for drag reducing fluids reported in this investigation, and also those reported by Spangler[2] and Wells[1] agree well with the trends obtained with a hot-film wedge probe and constant temperature anemometer. This is as should be expected since normal and shear stress effects on the piezoelectric probe in the center of the tube are small.

4) The 0.028 and 0.05% PIB solutions reported here showed some indication of viscoelastic effects near the wall. A direct comparison of the latter data with hot-film anemometry data on the same solution shows increasing error in the piezoelectric probe data as the wall is approached. It is believed that the error is not due to erroneous response of the piezoelectric probe, but to significant contributions of normal and shear stress effects.

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